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Network effects in Schelling's model of segregation: new evidence from agent-based simulation

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Abstract. According to two recent studies, Thomas Schelling's model of segregation is only weakly affected by the underlying spatial structure whatever its complexity. Such a conclusion is important from an urban planning perspective as it suggests that only a very restricted range of possible actions, if any, would be able to contribute to limiting social segregation, unless individual preferences are significantly modified. My own simulations show that, using appropriate graph-based spatial structures, one can reveal significant spatial effects and thus provide alternative planning insights. Cliques in networks indeed play a significant role, reinforcing segregation effects in Schelling's model. Introducing a small amount of noise into the model permits us to reveal this effect more precisely, without modifying the global behavior of the initial model. Furthermore, I show how a logistic model describes in a concise but precise way this global behavior at an aggregated level.

Keywords: agent-based modeling, cliques, networks, Schelling, segregation

1 Introduction

Thomas Schelling's (1969; 1978) model of social segregation is certainly one of the most debated models in the social sciences.(1) Imagine two colored groups of individuals, living on a chessboard city, and having the capacity to change their place of living according to the color of their neighbors. Schelling demonstrated that, if individuals have a mild preference for living near people of their own color, and if they move to satisfy their preference,(2) complete segregation at the city scale may occur.

Its capacity to demonstrate, in a simple and elegant way, how interdependent but noncoordinated local residential decisions may lead to unexpected social segregation surely explains the large success of this model. While scientists from various disciplines, ranging from social sciences (Clark and Fossett, 2008; Fossett and Dietrich, 2009; Laurie and Jaggi, 2003) and economics (Fagiolo et al, 2007; Pancs and Vriend, 2007) to physics (Dall'Asta et al, 2008; Gauvin et al, 2009; Stauffer and Solomon, 2007; Vinkovic and Kirman, 2006) and mathematics (Pollicot and Weiss, 2001; Zhang, 2004) handled this model, trying to find links with their own concepts and models, their conclusions all converged to Schelling's initial finding: segregation may occur at the city level even if every single agent is tolerant enough (mild preference) to accept an integrated pattern.

According to two recent studies (Fagiolo et al, 2007; Fossett and Dietrich, 2009), Schelling's model of segregation is so robust that it is only weakly affected by the underlying urban structure: whatever the size and shape of city, this undesired emerging phenomenon will happen. According to other authors (Clark and Fossett, 2008),

(1) For example, an advanced search of JASSS text using "Schelling" as a keyword points out 83 references on the JASSS website.

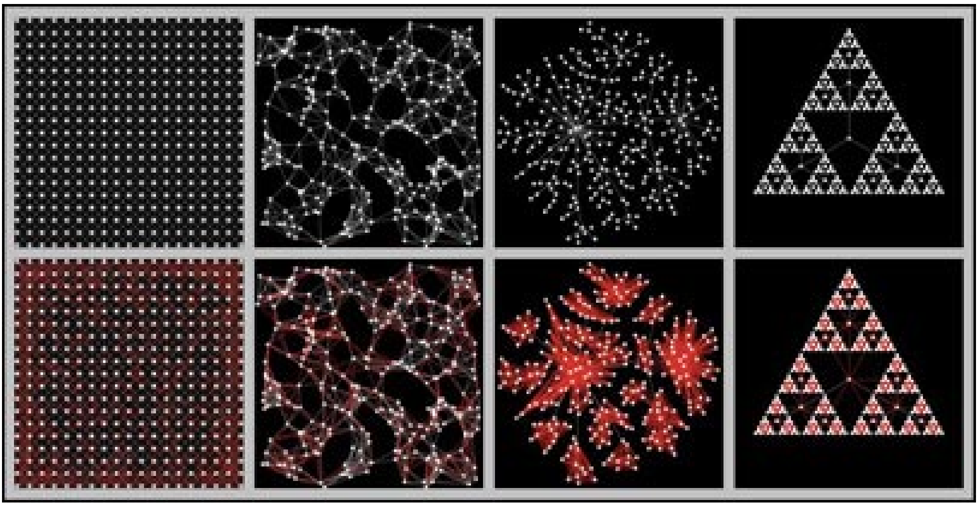
(2) Schelling's model is very limited in its scope as it is based only on individual preferences and thus does not take into account two other mechanisms at stake in social segregation, as identified long ago by sociologists: discriminative processes (occurring at the institutional and/or individual level) and socioeconomic factors.

individual preferences are evolving so slowly that no real improvement can be expected in the near future. Such conclusions may have important implications for public policy and urban planning as they suggest that only a very restricted range of possible actions, if any, would be able to contribute to limiting social segregation. However, the situation seems even worse than expected. Indeed, we can draw two complementary conclusions from our own simulations: actual and planned geometries of cities not only accelerate and reinforce Schelling's segregation processes but they may also favor intolerant behaviors. Therefore, fatalism should be banned: public policy and urban planning have their role to play in the increasingly urgent quest for sustainable cities.

2 Graph-based cities

The shape of an urban area and the way it is fed by the road network are two major ways that differences occur at the intraurban level. There are also generally very marked differences in accessibility between the center and the different suburbs. Moreover, certain areas or neighborhoods can be told apart by their relative impermeability: gated communities and ghettos can thus be defined as cliques, which in network theory jargon describe subsets made up of adjacent nodes. What can be demonstrated is that network shape in general and cliques in particular may play a significant role in the dynamic of Schelling's model, cliques acting as local attractors, or segregation 'traps'.

Let us illustrate this point with four increasingly hierarchized urban networks: (a) regular (grid), (b) random, (c) scale-free,[(3)] and (d) fractal [Sierpinski tree (figure 1)]. These last two networks may appear a bit 'exotic' to the nonspecialist but in fact they share more similarities with real cities than regular or random networks. For example, Jiang (2007) demonstrated on a large sample of forty US cities of different sizes that urban street networks based on street–street intersection display a scale-free property. Furthermore, fractal patterns are often presented as relevant for urban systems, when it comes to imagining more livable and sustainable cities (Batty, 2008; Frankhauser, 2008).



(a) (b) (c) (d)

Figure 1. [In color online.] (a) Regular, (b) random, (c) scale-free, and (d) fractal (Sierpinski) networks (top) and their corresponding neighboring graphs (bottom) for a fixed degree ($d = 10$; neighborhood defined using shortest-path algorithm).

(3) Generated following preferential attachment rules as proposed by Barabási and Albert (1999). See Song et al (2006) for evidence on the nonfractal topology of such networks.

These four networks could have been compared directly (as in Fagiolo et al, 2007), but I believe this would have introduced a severe bias linked to the varying distribution of degree. Indeed, as evaluation of preferences by each agent is based on the proportion of his n occupied neighbors having a given color, the de facto weight of each neighbor is $1/n$ and therefore small neighborhoods are more sensitive to changes than larger ones. To avoid this problem, the number of neighbors is fixed arbitrarily: each node is connected to its n closest nodes, as defined by a Floyd routing algorithm (Floyd, 1962). The resulting neighboring graphs obtained then present an interesting effect: they are more or less marked by the existence of cliques.

As the number of neighbors n is constant, each resulting network can be characterized by a simple clustering coefficient, C_i [4] (Watts and Strogatz, 1998):

$$C_i = \frac{2E_i}{k_i(k_i - 1)}, \qquad 0 \leqslant C_i \leqslant 1,$$

where E_i is the number of connected pairs among neighbors of node i and k_i is the degree of node i. This indicator varies between 0 (no connected pairs among neighbors of node i) and 1 (fully connected neighbors). Averaging these local values provides a global indicator which is very useful when comparing different network structures:

$$\bar{C} = \frac{1}{n}\sum_{i=1}^{n} C_i .$$

The four networks characterized by a fixed degree (10 neighbors), present different clustering values, as expected from their neighboring graphs (table 1).

Table 1. Properties of the four networks and their neighboring graphs (degree = 10).

Property	Network			
	grid	random	scale-free	Sierpinski
Number of nodes	361	361	361	363
Number of edges (structural graph)	1332	1448	360	364
Number of edges (neighboring graph)	2039	1749	1960	2134
Clustering (neighboring graph)	0.55	0.65	0.99	0.95

3 Schelling's agents

According to Schelling's model, each node of these graphs may be seen as a possible living place, occupied (or not) by an agent belonging to a given category. It is well known, however, that the density of agents needs to be neither too low nor too high for the system to become self-organized (Gauvin et al, 2009; Vinkovic and Kirman, 2006). Too many vacant nodes would limit the contacts between agents, keeping their spatial distribution random whatever their individual preference level, while too few free nodes would just 'freeze the system'. For each generated network a fixed proportion (80%) of nodes is then randomly populated with a corresponding number, m, of agents, half of them being of color x and the remaining part being of color y. Each agent is able to identify his neighborhood composition: ie, the proportion, P_{ij}, of his occupied neighbors being unlike him. At each time step each randomly selected agent A_i computes his utility, defined as a step function:

$$U_i = \begin{cases} 0, & \text{if } P_{ij} > \lambda, \\ 1, & \text{if } P_{ij} \leqslant \lambda, \end{cases}$$

[4] See Newman et al (2001) for a discussion on the limitation of this indicator when the degree varies and their proposal for a more robust indicator.

with λ a tolerance threshold value, embedding the agent's preference. Agent A_i then moves if he feels unsatisfied with his current location ($U_i = 0$) and if there is at least one vacant node—randomly chosen if there are more than one—allowing the agent to increase his utility ($U_i = 0 \rightarrow U_i = 1$).

For various possible values of the tolerance parameter[5] $\lambda = \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$, 1000 simulations were achieved and a social mixity index calculated. Following Schelling himself, this index is simply the mean proportion of contacts between unlike neighbors (omitting empty nodes):

$$M = \frac{1}{|A|}\sum_{i=1}^{|A|} P_{ij} \, ,$$

where $|A|$ is the number of agents (cardinal). This index then varies between 0 (complete segregation) and approximately 50% (mixity). It may be noticed that a 50% mixity can be obtained either from a random distribution of agents (the most probable and stable result) or from a more integrated 'chessboard'-like pattern (a highly improbable and unstable result).

A simulation ends if one of two conditions is respected: either the system converges towards equilibrium or the simulation time exceeds a given threshold value. Equilibrium is obtained when each agent is satisfied with his current location, that is:

$$\sum_{i=1}^{m} U_i = |A| .$$

It may be noticed that static patterns (ie, no agent moves any more) can be obtained even if no equilibrium is reached:

$$\sum_{i=1}^{m} U_i < |A| .$$

Gauvin et al (2009) call such state 'frozen'.

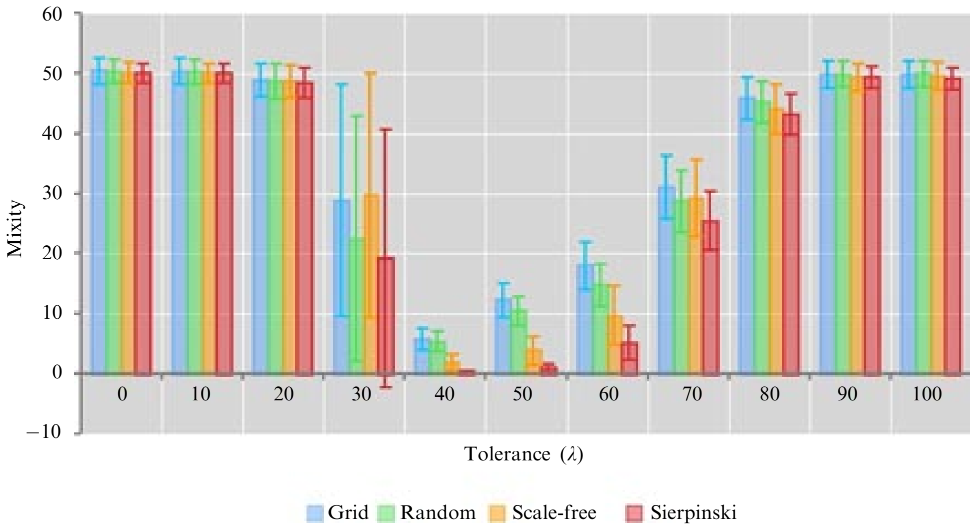


Figure 2. [In color online.] Average mixity index and its standard deviation (1000 simulations), for each of the four networks.

[5] Parameter λ is a proportion, expressed as a percentage, as it refers to a threshold value of P_{ij}, defined for each agent A_i as the proportion of occupied neighbors being unlike him.

Results (figure 2) confirm the robustness of Schelling's model but also stress significant discrepancies between the various networks, thus suggesting the possible role of network hierarchy and cliques in Schelling's segregation process.

4 Robustness of Schelling's model and cliques effect

The typical three phases (Dall'Asta et al, 2008; Gauvin et al, 2009) of Schelling's model are clearly identified, whatever the network structure, demonstrating the robustness of this model:

(1) convergence towards mixed states for high tolerance values ($\lambda > 70$);

(2) convergence towards increasingly segregated states for a large range of tolerance values ($30 < \lambda < 70$);

(3) absence of convergence for lower tolerance values ($\lambda < 30$).

For high tolerance values ($\lambda > 70$) the system rapidly converges (ie, reaches equilibrium) towards a mixed state. Then, a dramatic change occurs for a slight decrease in tolerance ($\lambda = 70$): while still being tolerant at an individual level (preferences), by their moves, agents make the global system converge towards a segregated state. As tolerance decreases ($30 < \lambda < 70$), highly segregated states are obtained. For a mild preference value ($\lambda = 50$) every simulation converges from an initial random distribution to a highly segregated pattern, for each of the four generated networks (figure 3).

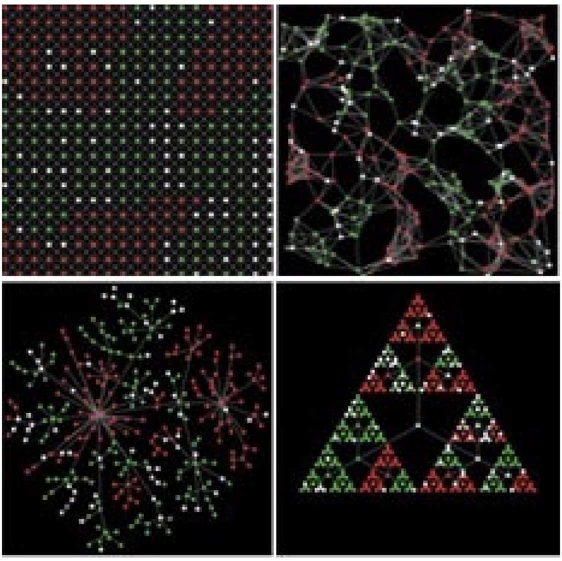


Figure 3. [In color online.] Examples of equilibria attained for mild preferences ($\lambda = 50$, density = 80%, degree = 10, white squares = vacant modes).

Although having a mild preference, each agent ends up in a highly segregated community, with few or no contacts with unlike agents. Patterns are clearly related to the neighboring graphs (figure 1), underlying the importance of cliques as 'segregation traps'. Observation of the model dynamics confirms the role played by cliques, but also the locking-in role of 'entry' nodes (figure 4). Cliques are rapidly filled up by an agent population of one or another color. As their connection with the rest of the network is limited to a reduced number of nodes (called 'entry' nodes), we can assume surface tension to be concentrated over this limited subset of nodes, thus reinforcing local anchorage of growing clusters. Network topology in general and cliques in particular

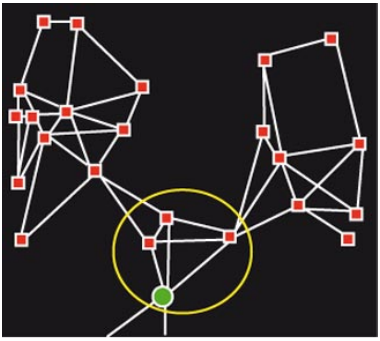


Figure 4. [In color online.] Cliques as segregation traps and the locking-in role played by 'entry' nodes (degree = 10; neighboring links not represented here).

seems to play an important role in the dynamic of the whole model, by channeling agents towards local attractors. Once these pockets of local order become sufficiently filled in, agents are then 'protected' from exogenous perturbations: no unlike agent will settle down any more. This process may be compared with the one described by Keeling (1999) for epidemics, who showed how cliques reduce the initial spread of an epidemic, but also the final proportion of the population that the epidemic reaches.

Decreasing tolerance ($\lambda = 30$) then leads to a second phase transition: agents find fewer and fewer vacant nodes corresponding to their preference and therefore wait for a better place to become vacant, while being unsatisfied with their current location. This transition area ($\lambda = 30$) is highly unstable and, depending on the initial configuration and the course of the simulation, 'frozen states' can be reached. In such situation the model does not converge towards equilibrium: while no agent can move any more, a varying proportion of them are not satisfied with their current location. The proportions converging towards equilibrium over 1000 runs, for this specific tolerance value ($\lambda = 30$ and 50 iterations)[6] vary from about 30% of the runs for the grid (31%) and scale-free (32%) networks, to 48% for the random network, and 56% for the Sierpinski tree. For very low tolerance values ($\lambda = 30$), no equilibrium can be found, as agents are trapped in a global frozen state, waiting forever for better places (ie, increasing their utility) to become available.

Although each of the four networks follows this global trend, significant differences occur at the two transition areas ($\lambda = 70$ and $\lambda = 30$), and within the segregated state ($30 < \lambda < 70$). The two hierarchized and highly clustered networks (scale-free and Sierpinski) apparently contribute to reinforcing the segregation process, letting the system converge towards significantly[7] higher segregation states.

Increasing the size of networks only marginally affects this very robust result. Figure 5 displays the curves for three Sierpinski networks of increasing size (364, 781, and 1464 nodes). As can be seen, the curves are very similar, whatever the network size. However, increasing the degree (number of neighbors) significantly impacts the result.

[6] This threshold depends both on initial conditions (network size and shape, density of agents) and simulation parameters (tolerance). However, with the decision rules defined so far, the number of iterations needed to reach convergence (being defined either as an equilibrium or as a frozen state) is usually small. In the present case preliminary convergence studies revealed that fifty iterations is a relevant limit: if the system does not converge in fifty iterations, then its probability of finding such a state in more iterations is negligible. Introducing some noise in the system, as I do in the next section, increases this value by one order of magnitude.

[7] Using Student's *t*-test for comparison of sample means.

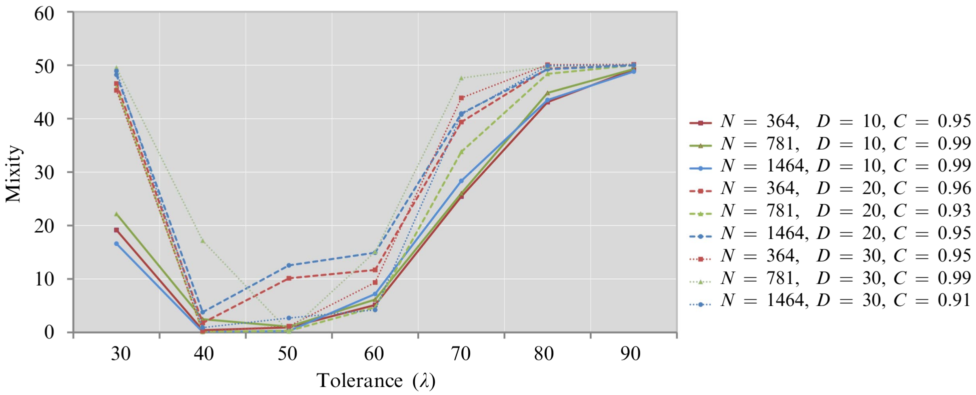


Figure 5. [In color online.] Comparison of results for three Sierpinski networks of increasing size (average mixity index from 1000 simulations. N is the number of nodes, D is the degree, and C is the clustering coefficient. (Confidence intervals are not shown, but are of the same order of magnitude as figure 2).

The segregation process is indeed retarded for high tolerance values ($\lambda = 70$) while frozen states are systematically obtained for $\lambda = 30$. As stated in section.2, the main explanation resides in the evaluation of preferences by each agent. Indeed, as it is based on the proportion of his n occupied neighbors having a given color, the weight of each neighbor is $1/n$ and therefore small neighborhoods are much more sensitive to changes than larger ones. Increasing the degree reduces the cascading effects leading to segregation for high tolerance values ($\lambda = 70$) while it decreases the possibility of agents finding a free node corresponding to their preference for low tolerance values ($\lambda = 30$). Such frozen states, easily obtained whenever one introduces more constraints in the system, suggest that adding some 'flexibility' in the system may be interesting.

5 Varying 'temperature' in Schelling's model

One could think of a physical analogy: by 'heating up' this system, we would be able to increase its entropy and therefore increase the range of possibilities for agents, as well as the range of possible states the system could reach. Adding some noise in the agents' decision rule allows such an objective to be reached. Several options are therefore available. For example, one can introduce the possibility that satisfied agents can also move, and not just restrict such a possibility to discontent agents. According to Gauvin et al (2009, page 296), "that rule introduces some noise in the dynamics and is useful to avoid a particularity of the original Schelling model, namely that the system may end up in states where the clusters are large but finite, so that strictly speaking no large-scale segregation occurs." Another option is to introduce the possibility of 'mistakes' (Zhang, 2004): agents who are willing to move may choose a destination which decreases their utility: for example, because they have incomplete or inaccurate information on the new context they face. It is this last option we follow here, by introducing a new parameter S (for 'noise'), ranging from 0 to 1 and interfering with the decision process in the following way.

For each unsatisfied agent $A_{i,\,U_i=0}$, we generate a random number, r, from a uniform distribution [0, 1] and use the following rule: if $s < S$, then agent A_i will move to a randomly chosen vacant node, whatever the expected utility value on that destination node ($U_i = 0 \rightarrow U_i = 1$ or $U_i = 0$). Introducing even a small amount of noise ($S = 0.1$) considerably 'fluidifies' the system in a very striking way (figure 6).

One can immediately see two interesting phenomena. First, introducing a small amount of noise in the model improves its capacity to reach equilibrium for low tolerance values ($\lambda < 40$). For scale-free and Sierpinski networks, convergence is systematic even when tolerance is null ($\lambda = 0$). Therefore, adding a small amount of noise transforms Schelling's model into an optimization algorithm, able to find spatial

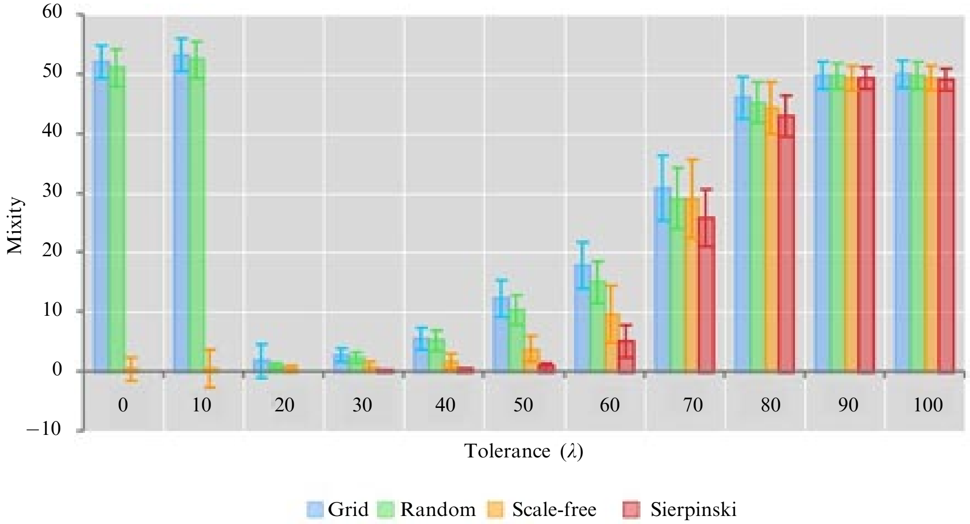


Figure 6. [In color online.] Average mixity index and its standard deviation (1000 simulations) for each of the four networks, with noise in the system ($S = 0.1$).

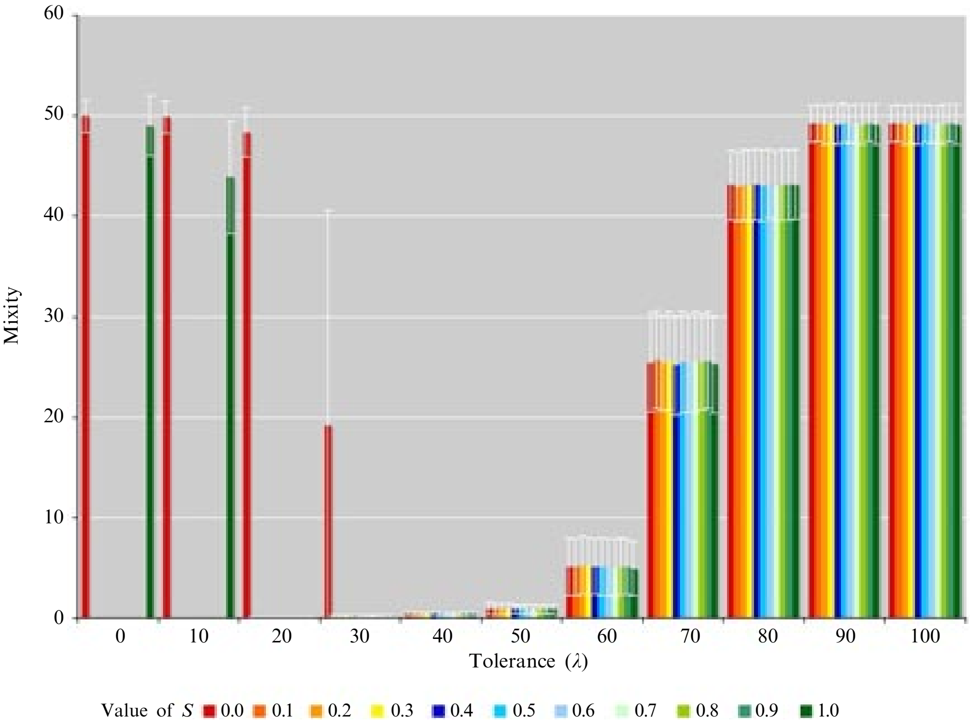


Figure 7. [In color online.] Mixity values reached for various noise levels, N^2, for the Sierpinski network (density of occupied nodes = 80%).

patterns respecting heavy constraints on parameter λ. The underlying topology reinforces this capacity, especially when networks are highly interconnected (presence of cliques). Furthermore, this capacity is extraordinarily targeted: the course of the model does not seem to be affected by this small amount of noise, unless the second phase transition is reached ($\lambda = 30$). Figure 7 clearly underlines this property for the Sierpinski network: whatever the level of noise added, results are not significantly affected, until $\lambda > 30$. Then, for $\lambda = 30$ and $\lambda = 20$, any amount of noise allows the model to reach equilibrium whereas the standard Schelling's model does not. As λ decreases further ($\lambda < 20$), a new state appears for $S = 1$ (ie, 100% of unsatisfied agents move to any vacant node whatever their expected utility there), characterized by never-ending movements of unsatisfied agents. It may be noted that this chaotic state appears only when $S = 1$, ie, when the system is populated with 'knee-jerk' agents, moving to any vacant node as soon as they feel unsatisfied. Therefore, preserving an even small amount of 'opportunist' Schelling-like agents is sufficient to let the system converge towards equilibrium.

6 Sensitivity to initial conditions and network effects

In this 'tipping' model, segregation is then characterized at a stochastically stable state that tends to emerge and persist in the long run regardless of the initial state (Zhang, 2004). However, depending on the initial configuration of agents, their random selection and the succession of events (path dependency), very different equilibria can be reached. In that sense, despite its global robustness, Schelling's model is also sensitive to initial conditions: different initial random patterns of agents may lead to very different final patterns. However, this sensitivity depends also on the value of the tolerance threshold used and its proximity to a stable or unstable zone of the parameter space. Indeed, as illustrated by figure 2, variability of final mixity index varies with tolerance value λ and reaches the highest values for $\lambda = 30$ and to a lesser extent for $\lambda = 70$ (phase transitions). Between these two values, final configurations obtained differ essentially in their details, as the global state—measured by mixity index—is quite stable. However, the situation is completely different when $\lambda = 30$. Indeed, as figure 2 suggests this zone of the parameter space is highly unstable and initial random

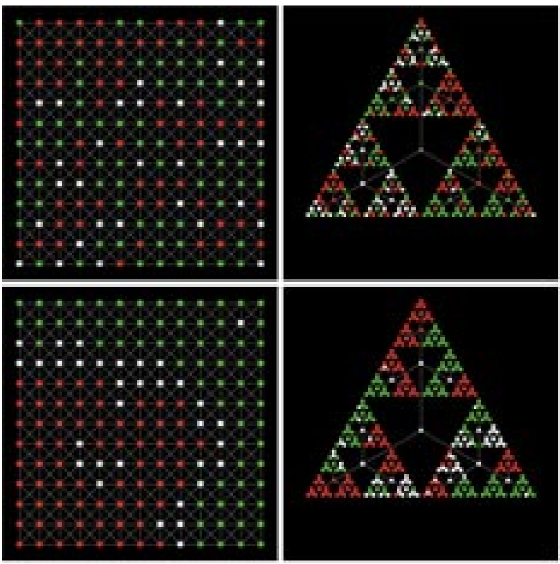


Figure 8. [In color online.] Example of bifurcation at work when $\lambda = 30$ (noise = 0; density = 80%).

distributions of agents can lead to very different configurations, depending on the capacity of the system to converge towards an equilibrium or to remain in a frozen state. Figure 8 illustrates this phenomenon, which can be seen as a bifurcation: from a given random initial distribution of agents and without added noise, the system may converge towards a completely segregated configuration (mixity $< 2\%$ for both networks) or remain in a frozen state characterized by a high mixity index (mixity $> 40\%$ for both networks).

Therefore, when $\lambda = 30$ Schelling's model is very sensitive to initial conditions and is consequently highly unstable, whatever the underlying network used. It may be noted that adding a small amount of noise allows this issue to be mastered, as figure 5 and previous developments show. However, in addition to these global characteristics and adding just a limited amount of noise, one can also reveal a specific behavior of the model in the Sierpinski network, characterized by a well-defined hierarchy of nodes. Indeed, as simulations point out (figure 9), the final state of the major central nodes after convergence depends closely on the tolerance value.

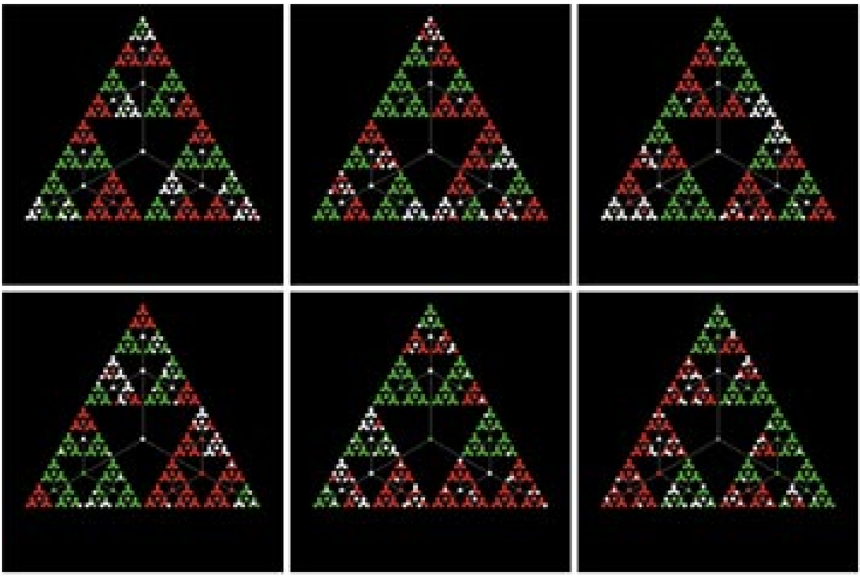


Figure 9. [In color online.] Influence of node centrality on its final state [noise = 0.1; density = 80%; λ varies from 0 (upper left corner) to 50].

At any time during the simulation, nodes are characterized by one of the three possible states (vacant; occupied by an agent of color x; occupied by an agent of color y). On a completely homogeneous network, it is hardly possible to predict the final value of a given node. However, for the Sierpinski network and for low tolerance values ($\lambda < 30$), this is no longer true; whatever their initial state, central nodes almost systematically converge towards the same final state (vacant), separating cliques of homogeneously colored agents. Therefore, for highly hierarchized networks such as the Sierpinski network, some nodes display very specific behavior depending on their topological situation. Space does make a difference, but at global and at local levels.

7 A global model

As figure 6 suggests, mixity can be defined as a step function of tolerance (λ), each underlying network having its own signature. Therefore, given the existence of two asymptotes (mixity $=$ 0 and mixity $=$ 50) and of acceleration and deceleration phases between them, a three-parameter logistic model may characterize this relation in a synthetic form:

$$g = \begin{cases} 50\,, & \text{if } \sum_{i=1}^{m} U_i < |A|\,, \\ \dfrac{\gamma}{1+\exp(-\alpha-\beta h)}\,, & \text{if } \sum_{i=1}^{m} U_i = |A|\,, \end{cases}$$

with $g =$ mixity and $h =$ tolerance (λ). Estimating parameters $\{\alpha, \beta, \gamma\}$ for each of the four networks leads to very tight fits ($R^2 > 0.99$). Figure 10 shows the various S-curves obtained.

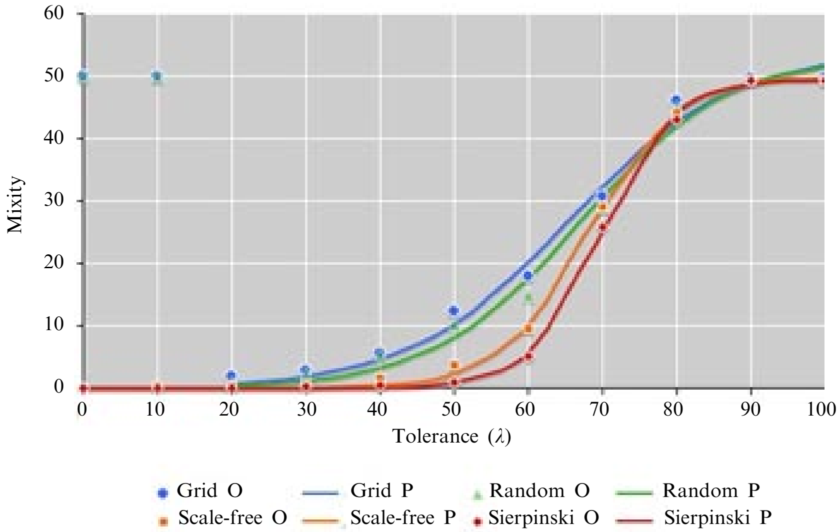


Figure 10. [In color online.] Logistic models fitted, for converging tolerance values [noise $=$ 0.1; in the legend 'O' is for observed (from simulation) and 'P' for predicted (model fitting)].

The parameters obtained (table 2)—and especially $\{\alpha, \beta\}$ as γ is a scale parameter—allow Schelling's model behavior to be compared in a very synthetic manner on various underlying structures. The surface between the two families of curves (grid and random versus scale-free and Sierpinski), coupled with the step-like structure of the logistic fits, give a precise idea of the capacity of the latter networks to accelerate and amplify Schelling-like segregation. These results suggest that, for a given underlying topology (network and degree) and by introducing a small amount of noise (with its conservative

Table 2. Parameters estimates and goodness-of-fit statistics (R^2) for each network.

Parameter	Network			
	grid	random	scale-free	Sierpinski
α	−6.064	−6.763	−11.319	−14.299
β	0.092	0.101	0.166	0.205
γ	54.019	53.632	50.076	49.406
R^2	0.991	0.992	0.999	0.999

properties suggested), we should be able to characterize and even anticipate Schelling's model behavior. On large networks, characterizing real cities, this could make a difference as the computation burden involved in simulation may exceed our capacities. Combined with further knowledge on local segregation dynamics related to cliques, these results might contribute to an assessment of urban planning policies.

8 Conclusion

So far, several conclusions can be drawn from these simulations. Firstly, and confirming previous work, Schelling's segregation process is robust and happens on very different underlying structures, even far away from the homogeneous grid initially used by Thomas Schelling. Secondly, this model is also very robust to random fluctuations. Indeed, introducing a small amount of noise in the model dynamic in the form of a small proportion of 'knee-jerk' agents, moving to any vacant node as soon as they feel unsatisfied whatever their expected utility, has a very limited impact on its global behavior. However, for specific tolerance values, such an add-on allows one to reach equilibrium solutions that were not attainable with the standard version. Under this slight modification, it was shown that the global behavior of Schelling's model can be formalized in a simple but precise way, with a three-parameter logistic model.

These fundamental properties of Schelling's model then allowed me to show by simulation that, contrary to previous conclusions (Fagiolo et al, 2007; Fossett and Dietrich, 2009), space does matter in this model. More precisely, network topology interferes with its dynamics, by accelerating and reinforcing its effects. Segregation occurs more rapidly on hierarchized networks characterized by cliques than on random and regular ones. Furthermore, such interconnected structures also authorize intolerant behaviors: equilibrium can indeed be reached with highly segregationists agents (tolerance weak or null), as soon as a small proportion of nonopportunist agents is introduced into the system. Preliminary investigations into 'real' cities confirm these theoretical results. Moreno et al (2009) applied the Schelling model to a large network (120 000 nodes), each building being embedded in an anisotropic neighborhood, defined using accessibility and graph-based algorithms, in the same spirit I did in the present work. Clique effects were also identified, though in a less systematic way due to the computational burden involved in the network size. These results may have important implications for public policy and urban planning as they suggest that actual and planned geometries of cities may not only accelerate and reinforce Schelling's segregation processes but may also contribute by favoring segregationist behaviors. Therefore, fatalism should be banned and laissez-faire should be fought: public policy and urban planning have their role to play. Limiting undesirable self-reinforcement processes may be possible and should be part of any ambitious urban politics.

Moreover, this work highlights the need for more integrated approaches to urban systems. Indeed, as Frankhauser (2008) shows, hierarchical and interconnected urban architectures prove useful in some aspects. In particular, fractal geometries, which may benefit cities and citizens, as they allow the maintenance of "a social mix by means of a higher local variability of densely and less densely populated zones and, on the other hand, to preserve huge empty zones in the neighborhood of urbanized areas, which may be imagined as natural reserves, agricultural zones or simply leisure areas offering rural amenities" (page 239). My simulations show evidence that such gains may be compensated by losses in terms of social mixity and integration. Based on these results, it seems obvious that more integrated approaches to urban systems should be encouraged and privileged in our increasingly urgent quest for sustainable cities. Among the challenges brought to the fore by this classical model, one finds new echoes: can we imagine cities' shapes and forms able to limit Schelling's segregation process?

Obviously, one cannot focus only on individual preferences, as suggested by Clark and Fossett (2008). Economic constraints, socioeconomic context, and precise survey-based knowledge of the precise mechanisms at work should be incorporated incrementally in the model, in order to gain new insights into its behavior in more complex situations, not limited to positive feedback and self-reinforcement processes. Carefully introducing heterogeneity into agent-based models, both at the level of agents' attributes and behaviors but also in the way we define agents' environment, is therefore a key issue the community should handle more vigorously, in order to improve our understanding of city dynamics.

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